Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

Topics: 1. More Related Rates   2. Warm up for Graphing.

1. *(from 2012 midterm2)* A ladder is leaning against a wall. The <u>bottom of</u> the ladder is being dragged away from the wall at a speed of 2 feet/second, and someone is extending <u>the ladder</u> at a rate of 1 feet/second. How fast is the <u>top of</u> the ladder moving when the ladder is 15 feet long, and when its bottom is at 12 feet from the wall?

x: distance of bottom from the wall $\frac{dx}{dt} = 2$

L: length of ladder $\frac{dL}{dt} = 1$

y: distance of top from the wall $\frac{dy}{dt} = ?$ $L = 15$ $x = 12$

$$x^2 + y^2 = L^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2L\frac{dL}{dt}$$

(with $x = 12$, $\frac{dx}{dt} = 2$, $L = 15$, $\frac{dL}{dt} = 1$)

$$y^* = \sqrt{L^2 - x^2} = \sqrt{15^2 - 12^2} = 9$$

$$2 \cdot 12 \cdot 2 + 2 \cdot 9 \frac{dy}{dt} = 2 \cdot 15 \cdot 1$$

$$\frac{dy}{dt} = -1 \ ft/s$$

2. For each of the following functions, determine on which intervals the function is positive or negative.
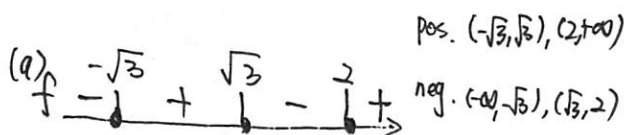
   (a) $f(x) = \frac{x^2 - 3}{x - 2}$
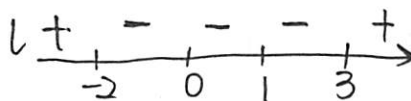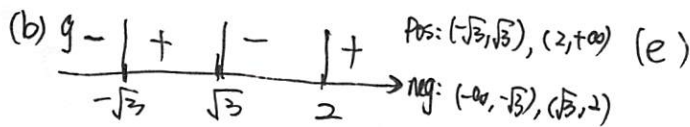
   (b) $g(x) = (x^2 - 3)(x - 2)$

   (c) $h(x) = 1 - \frac{4}{x^2}$

   (d) $k(x) = -\frac{1}{1 + 4x^2}$

   (e*) $l(x) = (x + 2)(x - 1)^2(x - 3)^3 x^4$

(a) f — number line with points $-\sqrt{3}$, $\sqrt{3}$, 2; signs $-, +, -, +$

pos. $(-\sqrt{3}, \sqrt{3}), (2, +\infty)$
neg. $(-\infty, -\sqrt{3}), (\sqrt{3}, 2)$

(b) g — number line with points $-\sqrt{3}$, $\sqrt{3}$, 2; signs $-, +, -, +$

Pos: $(-\sqrt{3}, \sqrt{3}), (2, +\infty)$
neg: $(-\infty, -\sqrt{3}), (\sqrt{3}, 2)$

(c) $h = \frac{x^2 - 4}{x^2} = \frac{(x-2)(x+2)}{x^2}$

h — number line with points -2, 0, 2; signs $+, -, -, +$

Pos: $(-\infty, -2), (2, \infty)$
neg: $(-2, 2)$

(d) $k(x) = -\frac{1}{1 + 4x^2} < 0$
negative on $(-\infty, +\infty)$

(e) l — number line with points -2, 0, 1, 3; signs $+, -, -, -, +$

POS: $(-\infty, -2), (3, +\infty)$
neg: $(-2, 3)$